ANSWERS

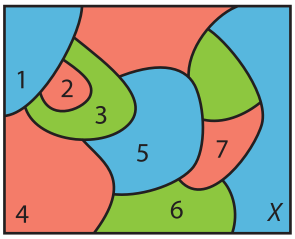
1. 139
2. blue
3. 7
4. CB*A
5. 81

SOLUTIONS

1. It should be clear that to keep the number as small as possible the first digit has to be 1. This leaves 12 to be shared between the other two digits. Again we want the second digit as small as possible and therefore we need to make the last digit as large as possible, namely 9, giving us 1?9.

   So far the digits add to 10 so the remaining 3 should be given to the second digit giving: **139**.

2. Working from the left to the right, the regions have to be coloured as shown so that no adjacent regions have the same colour. For example, region 3 has to be green as it is touching blue region 1 and red region 2. Then region 4 is red, region 5 is blue, region 6 is green region 7 is red, and region *X* is **blue**.

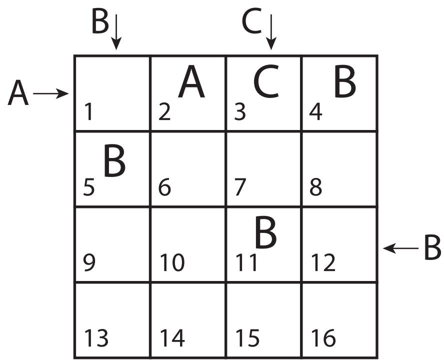


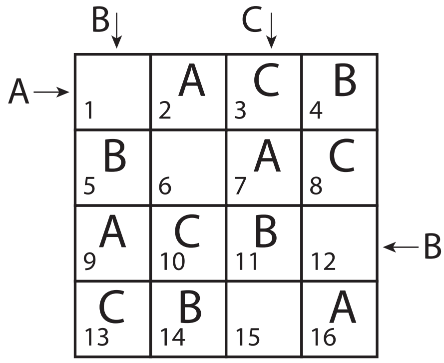
3. If all the 12 cycles were tricycles, then 36 wheels would have been used, which is too many by 5. Note that each time a tricycle is replaced by a bicycle one wheel less is needed. So we can reduce the number of wheels to 31 by replacing 5 tricycles with bicycles. We then have 12 – 5 = **7** tricycles (21 wheels) and 5 bicycles (10 wheels).

   Another approach is to find any numbers of bicycles and tricycles that use 31 wheels and then adjust it until we have 12 cycles. For example, we might start with 9 tricycles — using $3 \times 9 = 27$ wheels — and 2 bicycles to make use of the remaining 4 wheels. However 9 + 2 is not 12. Reducing the number of tricycles by one is useless as that releases 3 wheels that cannot be fitted to an integer number of bicycles. If we reduce the number of tricycles by 2 then we release 6 wheels, enough for 3 bicycles — we now have 7 tricycles and 5 bicycles, and 7 + 5 = 12 as required. So the number of tricycles is **7**.

4. Let's label the sixteen squares 1 to 16 as shown. Then we can argue as follows:
   * Square 1 must be empty, otherwise we cannot satisfy both the requirements that looking along the first row we must see A first, and looking down the first column we must see B first.
   * In fact since square 1 is empty there can be no further empty squares in the first row or first column. Consequently square 2 must be A, square 3 must be C (it cannot be B as the first non-empty square in column 3 must be C) and square 4 must be B.
   * Similarly, square 5 must be B.
   * The first B as seen from the right in the third row cannot be in square 12 as we already have a B in the last column, so it must be in square 11 (and square 12 must be empty). The first diagram shows progress so far.
   * By considering what letters need to be entered in each row and column, we can then conclude the following: square 10 is C, square 9 is A, square 13 is C, square 14 is B, square 8 is C, square 16 is A, and square 7 is A, as shown in the second, completed, diagram.

   The last row is therefore **CB*A**.





5. Initially one-tenth of the cars are yellow, so we can picture this using one box for the yellow cars and nine boxes for the other colours, all boxes having the same number of cars:

After the new car arrives the fraction of yellow cars increases, so this car must also be yellow. So the "yellow" box has grown by 1 car, as has the total. And now the yellow box is one-ninth of the total, so there are the equivalent of eight boxes for the other colours:

The number of cars that are not yellow is the same as before so that the nine old boxes and the eight new boxes must contain the same number of cars. But note that each new box has one more car than the old boxes. We can achieve this reorganisation by eliminating one of the old boxes and distributing the cars from it into the eight remaining boxes, putting one in each of them. This indicates that the old box must have contained 8 cars. Consequently, there were 80 cars originally (8 of which were yellow) and now **81** cars (9 of which are yellow).

An alternative approach would be to argue that the initial number of cars, call it N, must be a multiple of 10, while, after adding one more, the new number must be a multiple of 9. One can quickly run through $N = 10, 20, 30, \ldots$ and check whether $N+1 = 11, 21, 31, \ldots$ is a multiple of 9. Clearly $N = 80$ works, although we need to check that the number of yellow cars has increased by 1 as required — which it has as 80/10 =8, and **81**/9 = 9. (Other possible values of N, like 170, do not correspond to the additional 1 car.)

An algebraic approach would let y = number of yellow cars and n = total number of cars, initially. Then $y/n = 1/10$ and $(y+1)/(n+1) = 1/9$, from which $n = 10y$ and then $9y + 9 = 10y + 1$, giving $y = 8$, and $n = 80$.

PC 2015 **ANSWERS AND SOLUTIONS** **SET FOUR**

ANSWERS

1. 2
2. 14
3. 3
4. 199
5. 6

SOLUTIONS

1. Since the total \$17 is odd, there must be an odd number of \$5 bills used. Clearly one \$5 note and six \$2 dollar coins gives one way of paying (since $5 + 12 = 17$). The only other possibility is to use three \$5 notes and one \$2 coin (since $3 \times 5 + 2 = 17$). Therefore there are **2** different ways he can pay.

2. Since $140 = 10 \times 14 = 2 \times 5 \times 2 \times 7$, the only possible product of these factors which gives a number from 13 to 19 is $2 \times 7 = 14$, so the teenager is **14** (and the other two are aged either 2 and 5, or 1 and 10).

   Extension If instead the product of their ages was 330, how old would the teenager be?
   ($330 = 10 \times 33 = 2 \times 5 \times 3 \times 11$, so the teenager is $5 \times 3 = 15$.)

3. There are several possible approaches, but one is to notice that from row 1 and row 3 it is clear ♦ is 1 more than ✱. But then column 3 gives 2✱ + ♦ = 13 or 3✱ + 1 = 13, so 3✱ = 12 and ✱ = 4. Finally, from row 1, ♥ + 8 = 11, so ♥ = **3**.

   Alternatively, notice that the first column and the third row only contain the symbols ♥ and ♦:

   column 1 gives 2♥ + ♦ = 11 (i)

   row 3 gives ♥ + 2♦ = 13 (ii)

   Therefore 2 × (i) – (ii) gives 3♥ = 22 – 13 = 9, so ♥ = **3**, as above.

4. Numbering the first 9 pages 1, 2, ... , 9 uses 9 digits.
   Numbering the next 90 pages 10, 11, ... , 99 uses $2 \times 90 = 180$ digits.

   So far we have used $9 + 180 = 189$ digits, leaving $489 - 189 = 300$ digits. Now the remaining pages starting at 100, 101, ... are all 3 digit numbers, so there are $300 \div 3 = 100$ additional pages, making $99 + 100 = 199$ pages in total. Therefore the last page is **199**.

5. The sum of the lengths of the two equal sides is even, so since the perimeter 25 is odd, the length of the third side must also be odd. It could have length 1 unit in which case the two equal sides each have length $\frac{1}{2}(25 - 1) = 12$. That is $1 + 12 + 12 = 25$. It could have length 3 units with the two equal sides each of length 11, since $3 + 11 + 11 = 25$.

Similarly the third side could have length 5, 7, 9 or 11 with the equal sides respectively of lengths 10, 9, 8 or 7. For $5 + 10 + 10 = 7 + 9 + 9 = 9 + 8 + 8 = 11 + 7 + 7 = 25$, so each time the third side increases by 2 the equal sides decrease in length by 1. However the third side cannot have length 13 (or bigger), because $25 = 13 + 6 + 6$. However a basic property of a triangle is that the sum of the lengths of any two sides must be greater than the length of the remaining side, but here $6 + 6 < 13$. Therefore the third side has length 1, 3, 5, 7, 9 or 11 giving **6** possible isosceles triangles of perimeter 25 (with all sides of integer length.)